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Studies on the stress-strain relations of dual-phase steels

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The correlations of the work-hardening exponent, n, with quenching temperature, martensite volume-fraction (MVF) and solute concentration in ferrite are discussed and derived for dual-phase steel. The flow stress of dual-phase steel at low strain is suggested to be expressed by the combination of the terms due to plastic deformation in ferrite and elastic deformation of martensite. Previous experimental results are compared with the behaviour suggested by this theoretical work. In addition, an expression for the work hardening exponents at moderate strains and at the onset of necking are also theoretically suggested.

1. Introduction

Various theories have been developed in order to explain the mechanical properties, such as the stress–strain (σ–ϵ) relations, of metals with composite structure [1–7]. In general, they may be simplified into three types:

$$\sigma = K\epsilon^{n} \quad [1, 6-8], \qquad \text{Hollomon;} \quad (1)$$

$$\sigma = \sigma_0 + K\epsilon^{n} \quad [2, 5], \qquad \text{Ludwick;} \quad (2)$$

$$\sigma = \sigma_f V_f + \sigma_m(1 - V_f) \quad [3, 4], \qquad \text{Mixed Law,} \quad (3)$$

where, σ and ϵ are true stress and true strain, respectively, n is the work-hardening exponent, V_f is the second-phase volume-fraction, σ_f is the ultimate tensile strength of the second phase, σ_m is the stress carried by the matrix when the composite is strained to its ultimate tensile stress and K is a constant.

In the case of Hollomon-type behaviour (Equation 1), Morrison [1] found that in low-carbon steel, n is a single, or sometimes double, constant. In addition, n also depends on the microstructure and grain size of the material.

Moterio and Reed-Hill [5] concluded that in many cases the stress–strain relation can only be expressed by Ludwick's equation, Equation 2, instead of by Hollomon's equation, where σ_0 is the yield stress, and n and K may be either positive or negative.

For further modification, the mixed law was developed [3, 4].

In recent years, many studies have been conducted in order to produce a combination of all three theories to give a more complete application.

Generally, the equations developed for the description of the stress–strain relation are influenced to the greatest extent by the rigidity of the second phase in the composite metals [10, 11]. Since martensitic structure can always be plastically deformed [12], the martensite in dual-phase steel cannot be treated as a rigid body as previously reported. From the work of Ramos [13], the work hardening exponent, n, was already treated as a function of the carbon content in martensitic structures. The main purpose of this work is to discuss and to explain more completely and deeply the stress–strain relation based on the results of Ramos for dual-phase steels where the martensite is not treated as a rigid body.

2. Derivation and discussion

In recent years, from Equation 1, Davies [7] found that n is a function of martensite volume fraction (MVF) only. On the contrary, Ramos [13] has shown that the stress–strain relation cannot be simply expressed by this Hollomon-type equation. Ramos showed that n is linearly dependent on the carbon content in martensite through plotting the

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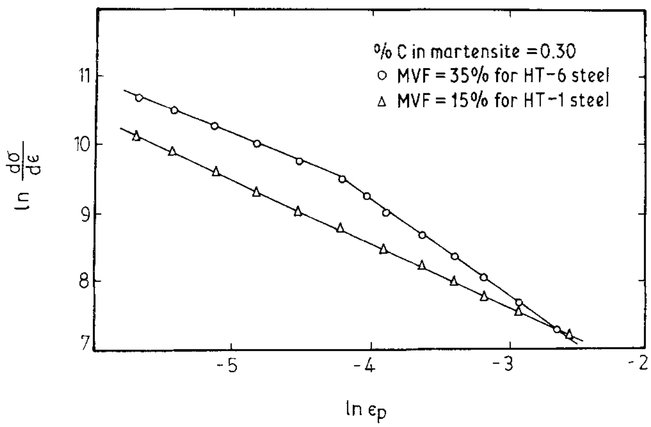

Figure 1 Plots of ln (dσ/dε) against ln ε for steels with the same carbon contents in the martensite, showing nearly parallel slopes at low strain levels, from [13].

value of ln dσ/dε against ln ε. He also found that the concentration of carbon in martensite and the deformation behaviour of dual phases had an interactive effect only at low strain levels. Based mainly on this result, an attempt has been made to analyse the detailed correlations between stress, strain, n, MVF and solute in ferrite for dual-phase steel during plastic deformation.

2.1. Relation between n and solute content in ferrite

It is well known that the solute in the ferrite or in the martensite can be determined from the phase diagrams. The results of Ramos, as shown in Fig. 1, indicate that the stress–strain curves of two different MVF values with the same carbon content (0.3 wt % C) in martensite, show clear and nearly parallel appearance of correlation at low strain levels.

Generally Equation 2 can be written as

$$\ln \frac{d\sigma}{d\epsilon} = (n-1)\ln \epsilon + \ln (nK), \qquad (4)$$

where $(n-1)$ is the slope of the curve in Fig. 1. This relation shows that $(n-1)$ closely depends not only on the MVF but also on the solute content in martensite and in ferrite at low strain levels. Since the solute content can be determined from the phase diagram, and the differences in the stress–strain curve of 0.3 wt % C martensite which is shown in Fig. 1 is attributed to different MVF values, Equation 2, therefore, can be rewritten as

$$\sigma = \sigma_0(f_m) + k(f_m)\epsilon^n, \qquad (5)$$

where f_m is the martensite volume fraction, and σ_0 and k are functions of MVF. Usually, a dual-phase steel is composed of ferrite and martensite, and the later is always stronger than the former especially at the beginning of a tensile test. In this case, the dislocations move much more easily in the ferrite than in the martensite. Since the slip in martensite can only happen as the ferrite is work-hardened to a certain level, this leads to the explanation that the value of $(n-1)$ depends only on solute content in the region of low strain.

With regard to the influence of solute content on work hardening, there have been many reports published [14, 15]. From one of Ramos' [13] results, as shown in Fig. 2, it can be clearly seen

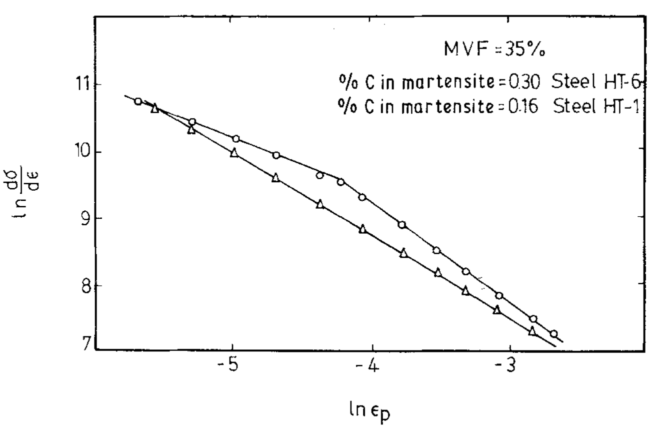

Figure 2 Plots of ln (dσ/dε) against ln ε for steels with the same MVF but different carbon contents in the martensite, showing non-parallel slopes at low strain levels, from [13].

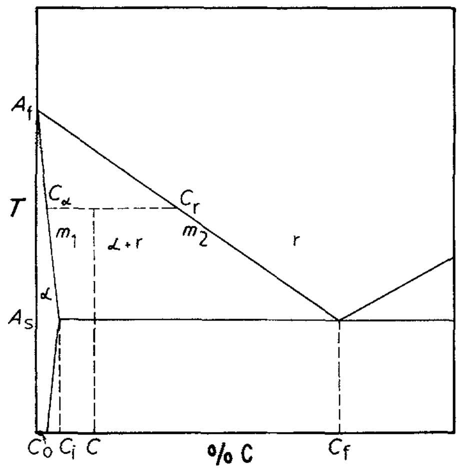

Figure 3 The modified phase diagram of Fe–C.

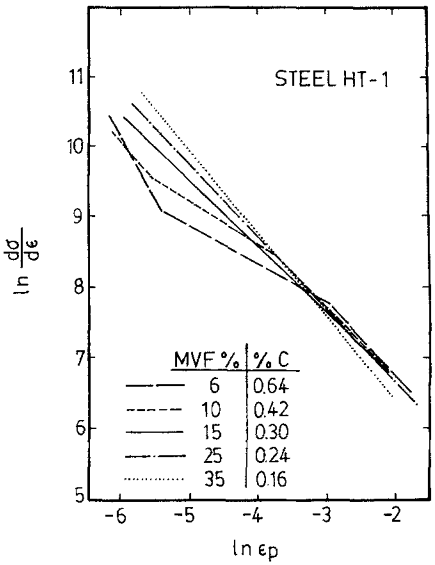

Figure 4 The relation between $\ln (d\sigma/d\epsilon)$ and $\ln \epsilon$ for different carbon contents in the martensite, from [13].

that the materials with different solute contents have different work hardening abilities.

On the basis of a slightly modified phase diagram of the iron–carbon system which is shown in Fig. 3, and the previous discussions, we may write the following equation to express the relation between n and the solute content in ferrite under equilibrium condition as

$$n = K_0 + K_1 C_\alpha, \qquad (6)$$

where K_0 is a constant which is influenced by the morphology of the martensite, the grain size and other precipitate particles and C_α is the solute content in ferrite.

By treating the boundaries in Fig. 3 as straight lines, C_α can be expressed as

$$C_\alpha = C_0 + (C_\mathrm{i} - C_0)\frac{A_\mathrm{f} - T}{A_\mathrm{f} - A_\mathrm{s}}, \qquad (7)$$

where C_0 is assumed to be the solute content that already existed as the alloy approaches the point of initiating the two-phase region from high temperature, C_i is the solute concentrations as indicated in Fig. 3 and A_s and A_f are the starting and finishing temperatures of austenite, respectively. Thus from Equation 4 we may write

$$\frac{\mathrm{d}\left(\ln \frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon}\right)}{\mathrm{d}\ln\epsilon} = n - 1, \qquad (8)$$

and from Equations 6 and 7 we get

$$\frac{\mathrm{d}\left(\ln \frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon}\right)}{\mathrm{d}\ln\epsilon} = a + b(A_\mathrm{f} - T), \qquad (9)$$

where $a = K_0 + K_1 C_0 - 1$ and $b = K_1(C_\mathrm{i} - C_0)/(A_\mathrm{f} - A_\mathrm{s})$. By using the results of Ramos, as shown in Fig. 4, and the existence of a linear function of temperature shown in Equation 9, it can be concluded that Equation 9 holds and is shown graphically in Fig. 5. It should be mentioned that the carbon content in martensite is always maintained lower than 0.4 wt% because otherwise plate martensite will appear with a drastic change in mechanical properties [16–19].

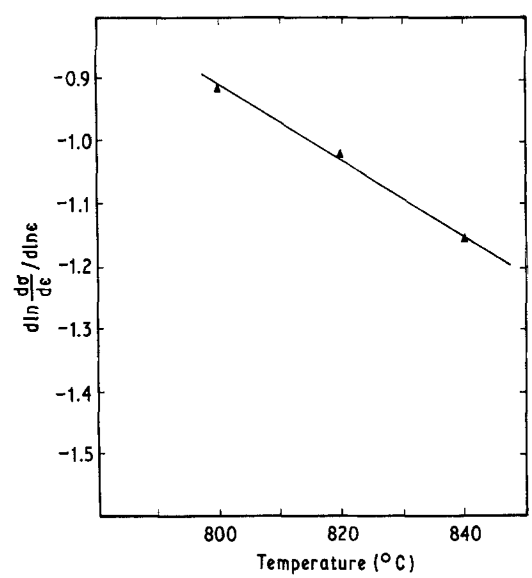

Figure 5 The slopes $(n - 1)$ varying with final quenching temperatures from [13] work plotted and compared with the theoretical predictions of Equation 9.

From the phase diagram, as shown in Fig. 3, the MVF value can always be determined by the temperature before quenching and the tie-law [20]. By using the triangle relation, n can be converted into a function of MVF,

$$f_m = \frac{C - C_\alpha}{C_r - C_\alpha} \tag{10}$$

and

$$\frac{C_r - C_0}{C_f - C_0} = \frac{A_f - T}{A_f - A_s}; \tag{11}$$

from Fig. 3 the slopes are

$$m_1 = \frac{A_f - T}{C_0 - C_\alpha} \tag{12}$$

and

$$m_2 = \frac{A_f - T}{C_0 - C_r}, \tag{13}$$

$$f_m = \frac{C - C_0 + \dfrac{1}{m_1}(A_f - T)}{\left(\dfrac{1}{m_1} - \dfrac{1}{m_2}\right)(A_f - T)} \tag{14}$$

and

$$A_f - T = \frac{C - C_0}{\dfrac{m_2 - m_1}{m_1 m_2} f_m - \dfrac{1}{m_1}}. \tag{15}$$

Therefore, from Equations 8 to 15,

$$n = a + \frac{e}{(m_2 - m_1) f_m - m_2} \tag{16}$$

where $e = K_1(C_i - C_0)(C - C_0) m_1 m_2 / (A_f - A_s)$.

By using Equation 16 the values of n with variation of the MVF may be calculated, shown by the solid line in Fig. 6. For comparison, data from the work of Ramos [13] are also plotted in Fig. 6. In another case, from the work of Davies [7] which is shown in Fig. 7, it is obvious that the broken line calculated from Equation 16 matches the experimental results more closely than the line calculated using the mixed-law equation developed by Mileiko [3] for relatively lower values of the MVF. It is suggested that when the MVF is greater than 40%, the martensite phase gradually becomes the dominant structure in the material and constricts the localized plastic deformation of ferrite. Thus, it is reasonable that Equation 16 fails to fit the experimental results when the MVF is greater than 40%.

It can be found that the ratios of m_1/m_2 obtained from Figs 6 and 7 are rather small in comparison with the value that is measured directly from a regular phase diagram of plain carbon steel.

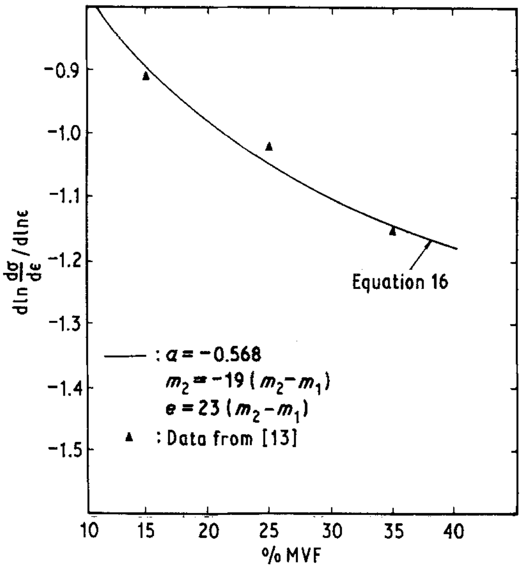

Figure 6 The slopes ($n - 1$) varying with the MVF from [13] plotted and compared with the theoretical predictions of Equation 16.

This is suggested to be due to the testing material with, firstly, a high manganese content austenite former and, secondly, a high silicon content ferrite former, which will make the values of m_1 and m_2 closer in the low-carbon concentration region.

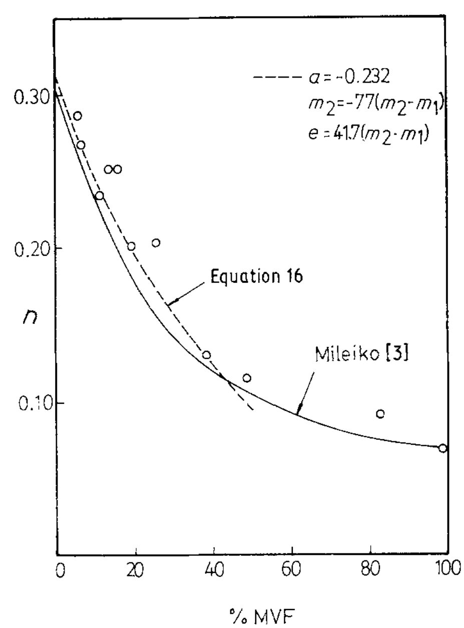

Figure 7 Experimental results of Davies [7] for the variation of n with MVF are plotted and compared with the Mileiko theory [3] and the theoretical predictions of Equation 16.

2.2. Suggestion of stress–strain relation at low strain levels

Dislocation movement is always easier in ferrite than in martensite, especially at low strain levels. Thus, it may be concluded that the work hardening takes place only in ferrite during the early stage of plastic deformation. It may therefore be assumed that

$$\sigma = (\sigma_{0\alpha} + K_\alpha \epsilon^{n\alpha}) f_\alpha + f_m C_m, \qquad (17)$$

where σ is the total flow stress, α indicates the content of ferrite phase, $\sigma_{0\alpha}$ is the yielding stress of ferrite, f_α is the volume fraction of ferrite and C_m is the static reinforcing effect of martensite. In Equation 17, on the right-hand side, the first term is due to ferrite work-hardening and the second term is due to static interaction between ferrite and martensite. In a more detailed consideration a term taking into account elastic deformation of martensite at certain strain is suggested to be required, giving

$$\sigma = (\sigma_{0\alpha} + K_\alpha \epsilon^{n\alpha}) f_\alpha + f_m C_m + f_m E_m (\epsilon - \epsilon_0) \qquad (18)$$

where E_m is the elastic constant of martensite and ϵ_0 is the elastic strain of martensite at yielding stress. Since the last term is usually much smaller than the other terms, especially during the early stage of strain, it can be combined with the second term, and Equation 18, for small strain with ϵ typically about 0.01, may be written as

$$\begin{aligned} \sigma &= \sigma_{0\alpha} + (C'_m - \sigma_{0\alpha}) f_m + (1 - f_m) K (\epsilon)^{\text{constant}} \\ &= K' f_m + \text{constant} \\ &= K' \frac{C - C_\alpha}{C_r - C_\alpha} + \text{constant} \\ &= K'' C + \text{constant}, \qquad (19) \end{aligned}$$

where C, C_α and C_r are functions of carbon content.

Values calculated from Equation 19 for $\epsilon = 0.01$ are compared with the experimental results of Davis in Fig. 8. If an extrapolation is made of the carbon composition of each steel to zero, it can be seen that all the values of flow stress converge at a point.

Comparing Equations 17 and 2 gives

$$\sigma_0 = \sigma_{0\alpha} + (C_m - \sigma_{0\alpha}) f_m \qquad (20)$$

$$K = (1 - f_m) K_\sigma \qquad (21)$$

$$n = n_\alpha. \qquad (22)$$

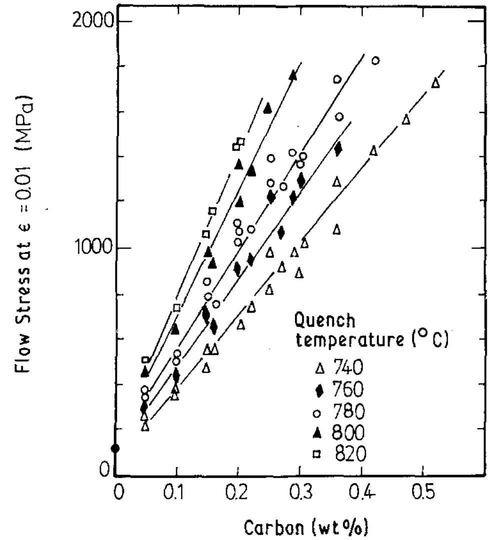

Figure 8 The flow stress at $\epsilon = 0.01$ plotted against carbon composition for different quench temperatures. Note that the curves converge at one point, [7].

These equations indicate that the work-hardening exponent, n, of a dual-phase steel at low strain depends only on the ferrite matrix. In addition, they show the linear dependence of yield stress with MVF provided by many previous workers [13] (see Fig. 9) and others [7, 14, 20, 21]. Equation 20 shows that when MVF approaches zero the yield stresses of steels with various carbon contents converge to a point, an observation also made by Ramos [13], and shown in Fig. 9.

2.3. Suggestion of a stress–strain relation at moderate strain levels and the relations among ultimate tensile strength, *n*, uniform strain and MVF

(a) As the work hardening increases to a certain level, plastic deformation of martensite begins. For

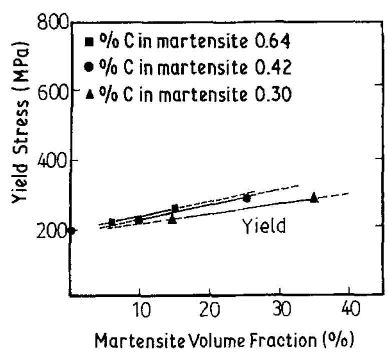

Figure 9 The linear relation between yield stress and MVF. The curves extrapolated to zero MVF converge at one point from [13].

this case, it may be assumed that the flow stress of a dual-phase steel may be written as

$$\sigma = f_\alpha(\sigma_{0\alpha} + K\alpha\epsilon^{n_\alpha}) + f_m[\sigma_{0m} + K_m(\epsilon - \epsilon_c)^{n_m}] + f_m C_m, \quad (23)$$

where ϵ_c is the strain as $\sigma = \sigma_\alpha f_\alpha + \sigma_{0m} f_m$.

If the strain is large enough and if the work-hardening exponent, n, in Equation 2 is single valued then, through Equation 23, it may be deduced that $n = n_\alpha = n_m$. This means that both the ferrite and martensite phases have the same work-hardening exponent. This suggestion remains to be checked by further experimental work.

(b) It is well known that the true stress and true strain have the relation, at the on-set of necking,

$$\frac{d\sigma}{d\epsilon} - \sigma = 0. \quad (24)$$

If Equation 2 holds and n has a single value, it may be deduced that

$$nK\epsilon_u^{n-1} - \sigma_{uts} = 0$$

$$\epsilon_u^n = (\sigma_{uts} - \sigma_0)/K$$

$$\frac{n(\sigma_{uts} - \sigma_0)}{\epsilon_u} - \sigma_{uts} = 0,$$

where ϵ_u is the uniform stress at the on-set of necking and σ_{uts} is the yield strain at ultimate tensile strength. Thus, finally, we may get

$$\sigma_{uts} = \frac{n\sigma_0}{n - \epsilon_u} \quad (25)$$

and

$$n = \frac{\sigma_{uts}\epsilon_u}{\sigma_{uts} - \sigma_0}. \quad (26)$$

For further proof of Equations 25 and 26 a more detailed experimental study is necessary.

3. Conclusions

Based on the assumptions for a dual-phase steel that: (a) grain size is a constant, (b) both phases show the same morphology, and (c) the transformation lines are straight, it may be concluded, both from derivation and previous experimental works, that:

(1) For the stress–strain relationship of a dual-phase steel

$$\sigma = \sigma_0 + K\epsilon^n,$$

where n can be expressed independently only by the temperature at the two phase region before quenching, the MVF or concentration of solute in ferrite matrix.

(2) The initial plastic deformation of dual-phase steel always begins at the ferrite matrix and the flow stress at low strain can be expressed by the combination of terms due to plastic deformation in ferrite, elastic deformation of martensite and static interaction between ferrite and martensite. This gives

$$\sigma = (\sigma_{0\alpha} + K_\alpha\epsilon^{n_\alpha})f_\alpha + f_m C'_m.$$

Based so far only on theoretical derivation it may be concluded that:

(3) Both the ferrite and martensite phases appear to have the same work-hardening exponent at moderate strain levels in the case of single-valued n that results from the following equation of flow stress

$$\sigma = f_\alpha(\sigma_{0\alpha} + K_\alpha\epsilon^{n_\alpha}) + f_m[\sigma_{0m} + K_m(\epsilon - \epsilon_c)^{n_m}] + f_m C_m.$$

(4) At the on-set of necking, the work-hardening exponent, n, of a dual-phase steel can be expressed as

$$n = \frac{\sigma_{uts}\epsilon_u}{\sigma_{uts} - \sigma_0}.$$

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